

# IMPERIAL COLLEGE LONDON

## Design Engineering MEng EXAMINATIONS 2018

For Internal Students of the Imperial College of Science, Technology and Medicine  
*This paper is also taken for the relevant examination for the Associateship or Diploma*

### Engineering Analysis EA 1.3 - Electronics

Monday 25<sup>th</sup> June 2018 10.00 to 11.30

## SOLUTIONS

*This paper contains TEN questions.*

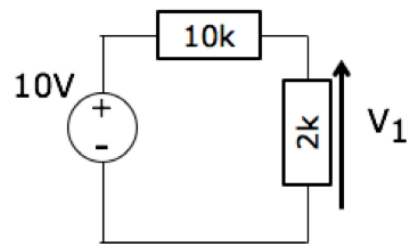
*Attempt ALL questions.*

*The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.*

**This is a CLOSED BOOK Examination.**

1. Find the value of  $V_1$  for the circuit shown in *Figure 1*.

[3]



*Figure 1*

### Solution

This question tests student's understanding of a simple voltage divider.

$$V_1 = 10V \times 2k / (10k + 2k) = 1.67V$$

Feedback: Almost everyone got this right! (average: 96%)

2. Figure 2 shows a circuit with three resistors R1, R2 and R3 connected as shown. The circuit has three terminals: A to R1, B to R2 and C to R3.

A 10V battery with a positive (+ve) and a negative (-ve) terminal is connected between a pair of terminals. Then a digital voltmeter is used to measure the voltage between another pair of terminals with following measurements:

- (i) +ve terminal to A, -ve terminal to C, voltage  $V_{BC} = 6V$ ;
- (ii) +ve terminal to A, -ve terminal to B, voltage  $V_{CB} = 2V$ ;

Given that  $R1 = 4k\Omega$ , calculate the values of R2 and R3.

[10]

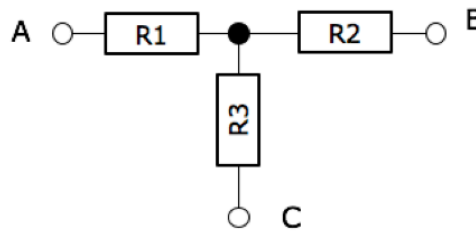


Figure 2

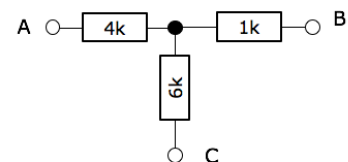
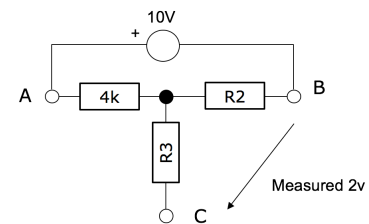
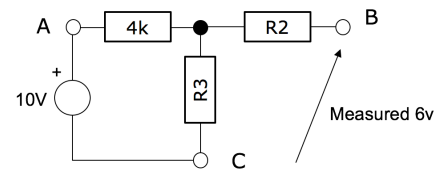
### Solution

This question is linked to Lab 2, Test 1. Given a star-connected 3-resistor network, how to find the resistor values from voltages. The solution is an application of voltage divider principle. The key is to realise that the measurements are done by voltmeter which has infinite impedance and draws zero current.

First use i): R3 and 4k divide 10v to give 6v, therefore  $R3/(R3+4) = 6/10$ . Hence  $R3 = 6k$ .

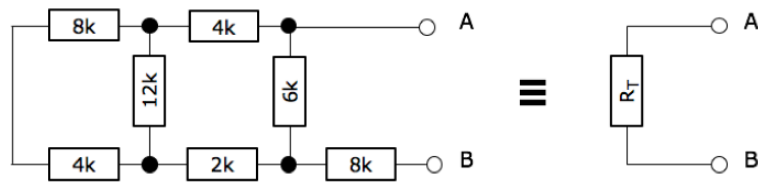
Use ii): R2 and 4k divide 10v to give 2v, therefore  $R2/(R2+4) = 2/10$ . Hence  $R2 = 1k$ .

Feedback: Most student could not do this question although it was part of Lab 2 challenges. Most forgot that measuring a voltage between, say B and C, to be 6v is NOT the same as connecting a 6v voltage source between the two terminals. Many wasted lots of time, then by apply KCL to try to solve for R2 and R3. (average: 52%)



3. For the circuit shown in *Figure 3*, derive the equivalent resistance  $R_T$  between nodes A and B.

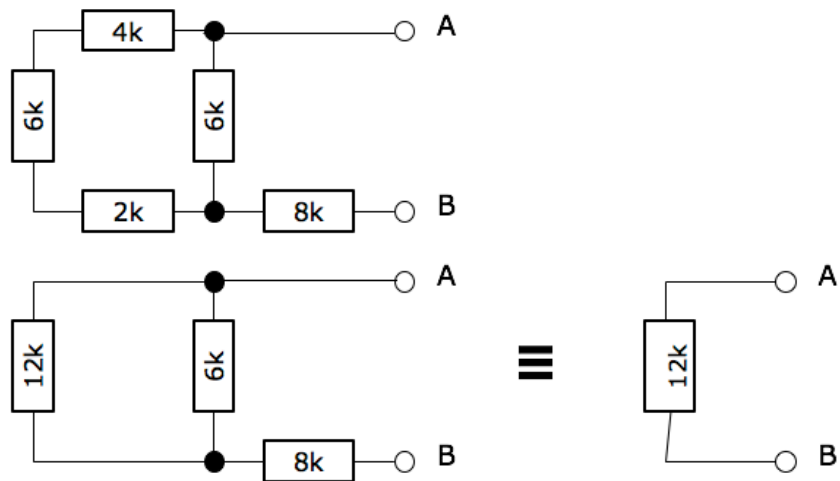
[7]



*Figure 3*

### SOLUTION

This question tests student's understanding of equivalent circuit and how to derive total resistance value when resistors are connected in series and in parallel.



Feedback: Most students got this one perfectly. (average: 97%)

4. When the car ignition is off, but the radio and headlights are switched ON, the car battery voltage at the terminals is measured to be 12.6V. When the ignition key is then turned ON, the battery voltage at the terminals drops to 10.6V. It is known that the radio and headlights together draw a current of 500mA, and the starter motor draws 100A during ignition.

Derive the Thévenin equivalent circuit for the battery.

[10]

## SOLUTION

This question tests student's understanding of the Thévenin theorem and how to apply this to a real-life situation.

$$V_{th} - I_B R_{th} = V_B$$

Therefore:

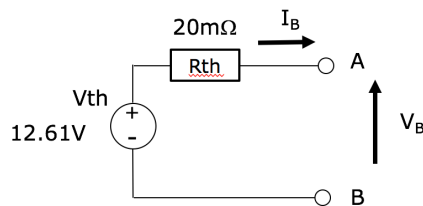
$$1: \quad V_{th} - 0.5R_{th} = 12.6$$

$$2: \quad V_{th} - 100.5R_{th} = 10.6$$

Eq 1 – Eq 2 gives:  $100R_{th} = 2$ ,  $R_{th} = 0.02\Omega$  or  $20\text{m}\Omega$

Substitute this back to Eq 1:

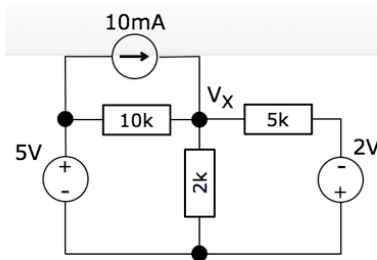
$$V_{th} - 0.5 \times 0.02 = 12.6, \text{ therefore } V_{th} = 12.61\text{V}$$



Feedback: Although this is a straight forward application of Thevenin equivalent circuits, students found this a hard question. Most common mistake is getting the sign of the IR drop across Rth the wrong way around, making  $V_B$  larger than  $V_{th}$ , which is of course impossible. (average: 61%)

5. *Figure 4* shows a circuit with three resistors, two voltage sources and a current source. Using the method of nodal analysis, find the value of  $V_X$ .

[10]



*Figure 4*

### SOLUTION

Apply KCL at node  $V_X$  (+ve current flowing out of node):

$$\frac{V_X - 5}{10} - 10 + \frac{V_X}{2} + \frac{V_X - (-2)}{5} = 0$$
$$\Rightarrow 8V_X - 101 = 0$$

Therefore,  $V_X = 12.63\text{V}$

Feedback: Most students can do this question. (Average: 81%)

6. The ASCII table is shown in the Appendix. Given that all numbers are represented as 8-bit values, complete the missing entries that are not shaded in the following table (*Figure 5*). (No marks will be awarded for this question unless you show how the solutions are derived.)

[8]

If the ASCII character is transmitted in UART format with 8-bit data and no parity bit, sketch the timing waveform of the digital signal for this character.

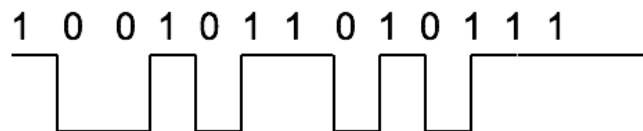
[4]

Hexadecimal	Binary	Unsigned Decimal	ASCII
5A			
		250	

*Figure 5*

### SOLUTION

Hexadecimal	Binary	Unsigned Decimal	ASCII
5A	<b>01011010</b>	<b>90</b>	<b>'Z'</b>
<b>FA</b>		250	



Feedback: Most students can do the first part well – converting between the different number representation. Many students did not draw the waveform for the start bit and the stop bit. (Average: 79%)

7. In the circuit shown in *Figure 6*, the switch is at position 1 (i.e. closed) for a long time before moving to position 2 (i.e. opened) at time  $t = 0$  sec.

- (a) What is the capacitor voltage  $v_C(t)$  at  $t = 0$  sec just before the switch is opened? [2]
- (b) What is the final value of  $v_C$  for  $t \rightarrow \infty$ ? [1]
- (c) What is the time constant of the function  $v_C(t)$  for  $t \geq 0$  sec? [2]
- (d) Derive the equation for  $v_C$  as a function of time  $t$ . [3]
- (e) Sketch the waveform of  $v_C(t)$  vs time  $t$ . [3]
- (f) If the  $30k\Omega$  resistor and the  $0.1\mu\text{F}$  capacitor are swapped around, and everything else being the same, sketch the waveform of  $v_C(t)$  vs time  $t$ . [5]

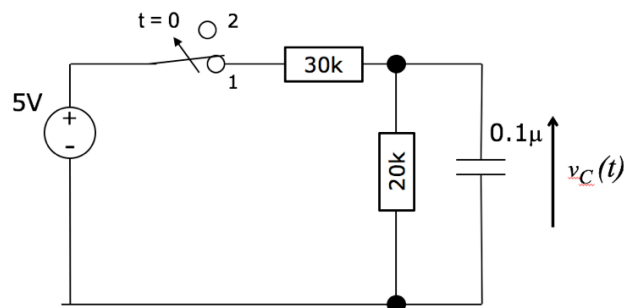


Figure 6

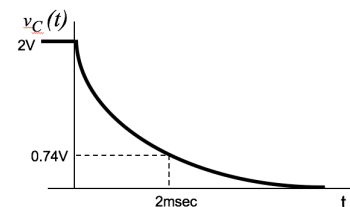
### SOLUTION

This question tests student's understanding of transient behaviour with RC circuits.

- (a) Capacitor will charge to the voltage across  $20k$  resistor, which is  $2V$ .
- (b) When switch is in position 2, capacitor will discharge to  $0V$  as  $t \rightarrow \infty$ .
- (c) In position 2,  $30k$  resistor is not in play. So time constant =  $20k \times 0.1\mu = 2\text{msec}$ .

(d) This is exponential decay:  $v_C(t) = 2(e^{-0.5 \times 10^3 t})$

(e)



- (f) If the  $30k$  and the  $0.1\mu\text{F}$  are swapped around, it becomes a CR circuit. When  $t < 0$  for a long time, and switch in position 1 the capacitor is charged to  $5V$  (positive to the left, the right terminal connected to the resistor is negative). At  $t=0$ , switch goes to position 2, and the circuit is now open – i.e. no current can flow. Therefore,  $V_C(t)$  is simply a  $5V$  constant voltage.

Feedback: Most student got a) and b). Only a couple of students got (f) correctly. (Average: 52%)



8. Figure 7 shows an inverting amplifier circuit constructed from an operational amplifier with a Gain-Bandwidth product of 1MHz that has a single power supply at 5V, two resistors R1 and R2, and a voltage source at 2.5V.

- (i) Write down an equation for the Gain of this amplifier. [2]
- (ii) Given that the amplifier has a gain of -14 and  $R1 = 2.2k\Omega$ , determine the value of R2. [2]
- (iii) A design is required to provide an overall gain of -200 for a signal with frequency component from 1kHz to 50kHz. Explain why the circuit in Figure 7 is unable to achieve the specification. [2]
- (iv) Hence or otherwise, design an additional circuit using a second operational amplifier to meet the required specification. [6]

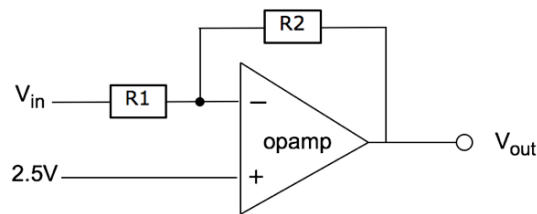
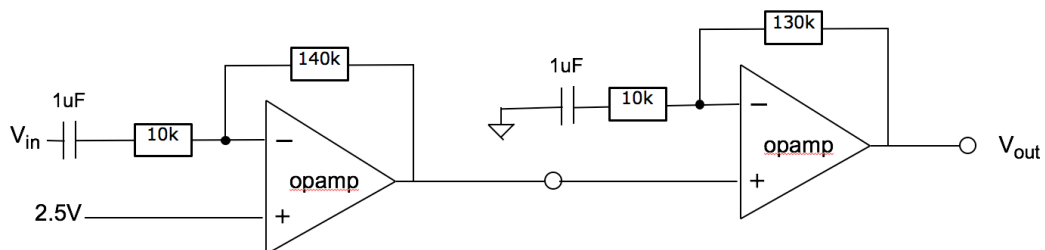


Figure 7

### SOLUTION

- (i) 
$$V_{out} = -\frac{R2}{R1}(V_{in} - 2.5) + 2.5$$
- (ii) For a gain of -14,  $R2 = 14 \times R1 = 30.8k\Omega$
- (iii) The gain-bandwidth product is 1MHz. Therefore at 50kHz (max signal frequency), the maximum gain is only  $\times 20$ . Furthermore, the amplifier is DC coupled. It needs to add a DC blocking capacitor at the input.
- (iv) The 1kHz lower frequency requirement define the capacitor coupling we need. With circuits provided, the lower corner frequency is compatible with the 1kHz lower limit. See circuit below:

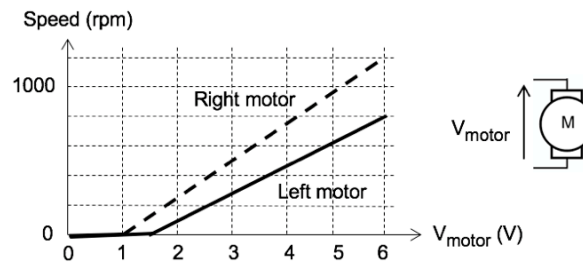


Feedback: Many students made mistake with (iv). (Average: 59%)

9. *Figure 8* shows the relationship between the speed of a DC motor (in revolutions per minute or rpm) and the average voltage across the motor windings for the right and left motors of a two-wheel vehicle.

(i) If both motors are driven with identical PWM signals with a high voltage of 6V and a 75% duty cycle, what are speed of the two motors? [5]

(ii) It is required that the vehicle be driven in a straight line. Explain how you might achieve this in your speed control algorithm for the minimum to the maximum speed. [5]



*Figure 8*

## SOLUTION

(i) From the graph, we can derive these equations for the straight lines:

$$\text{Right motor speed} = 240 \times V_R - 240 \quad \text{for } V_R > 1V$$

$$\text{Left motor speed} = 178 \times V_L - 267 \quad \text{for } V_L > 1.5V$$

75% duty cycle of a 6V PWM signal has an average DC value of 4.5V. From the equations, right motor speed is 840 rpm and left motor speed is 534 rpm.

(ii) To have both motors running at the same arbitrary speed  $S$ , we use the above equations to derive the values of  $V_R$  and  $V_L$  for the same  $S$ , and convert that into PWM duty cycle of each of the motors.

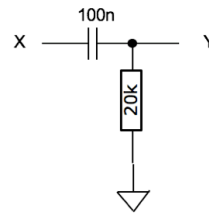
**Feedback:** Only a small proportion of students approach this analytically. Many just read this of the graph with large errors for (i). As a result, many did not give a good answer for (ii). (Average: 51%)

10. For the circuit shown in *Figure 9*,

(i) Write down the impedance of the capacitor. [2]

(ii) Derive the frequency response  $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$ . [6]

(iii) What type of filter is this circuit? [2]



*Figure 9*

### SOLUTION

(i) The impedance of the capacitor is  $\frac{1}{j\omega C} = -j\frac{10^7}{\omega}$ .

(ii)  $\frac{Y(j\omega)}{X(j\omega)} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega/500}{1 + j\omega/500}$

(iii) This is a high pass filter circuit.

Feedback: Many could not do this question, but those who did, generally got it correct.  
(Average: 50%)